

## **KARMA - Volume II**

An Actuarial Analysis of Regression to the Mean in Sports Betting Markets

## Table of Contents

Abstract.....	3
Hypothesis.....	4
Methodology.....	6
Data and Limitations.....	9
Results.....	10
What's New in Volume II.....	12
Appendix.....	13

## ABSTRACT

KARMA is a one-of-a-kind actuarial analysis of the major sports betting markets using data stretching back between one and two decades for each sport. KARMA is particularly unique in that it never predicts a score for any game. Instead, it capitalizes on inefficient markets by predicting *when* the line (i.e. the collective wisdom of the crowd) will provide value for an underperforming team by failing to incorporate the inevitable regression to the mean. The results indicate that regression to the mean is not adequately incorporated into sports betting markets and, subsequently, profitable opportunities exist.

KARMA includes a publicly available Test Center for users to see where value exists and how it varies amongst the major markets. Readers are encouraged to use the free Test Center and purchase the full underlying workfile with exact calculations and transparent results for every game in the dataset.

The Results and Appendix sections provide summary and detailed results for each market. Other inputs may yield better results and the optimal inputs/outputs may evolve over time depending upon the information each user decides is most predictive for future expectations. KARMA empowers gamblers to determine the appropriate risk tolerance and strategy based upon results in the Test Center. Any questions or comments should be directed to [karmatalktous@gmail.com](mailto:karmatalktous@gmail.com).

## HYPOTHESIS

The foundations of the hypothesis underlying KARMA rests on a few assumptions. First, the closing line represents the collective wisdom of the crowd. Each closing line represents the culmination of a sportsbook's best opening line (after considering team strengths, injuries, expected public interest in each team, and any other pertinent factors) as well as any changes in the line due to new info and bets.

The second important assumption emerged from the field of psychology; humans have, both individually and collectively, a strong inclination to detect patterns and assign causality where little or none may exist. This leads to us all routinely overvaluing the information available from the data provided by small samples. More data does *not* always mean more useful information. The mistakes of a) systematically overweighting sample sizes and b) discounting regression to the mean have been demonstrated numerous times in varying contexts, ranging widely from the work of [Amos Tversky](#) and [Daniel Kahneman](#) to a great book by [Gary Smith](#). Put simply, we as a species believe too confidently that what has already happened is more likely to happen again because it is easier for us to imagine; we've already seen it. It is often called a recency or an availability bias.

The combination of these key assumptions is what creates the KARMA hypothesis. KARMA is built on the hypothesis that ***betting on teams to regress towards efficiency in a sports betting market is capable of yielding consistently profitable results***. It is a simple idea to articulate though a more difficult one to test. In order to test the idea, several items need to be defined.

First: what is efficiency? And can we measure it?

Fortunately, sports betting minimarkets (the term used by KARMA for an individual game's short-lived betting market), in contrast to the stock market, have a defined end point. Each minimarket closes when the game starts and results are realized by all parties when the game ends. In the stock market, there is no universal definition of efficiency since the market exists into perpetuity. No *universal* settlement value is routinely realized for a company's stock. If there were universal agreement on a stock price, it would cease to trade until new information arises.

In contrast, sports betting markets are efficient if each competing variable - favorites vs underdogs, home vs away, overs vs unders - wins proportionately. Thus, winning 50% Against the Spread (ATS) is the universal measure of market efficiency at the end of a team or sport's season. Teams that won less than half their games across some variable (home games, favorite games, overs, etc.) can be said to have underperformed relative to expectations and vice versa. For Moneyline bets, we only need to make a slight calculation adjustment to measure Return on Investment (ROI) instead of record ATS. In a typical "bet 11 to win 10" market, the efficient Moneyline ROI is equivalent to the house edge of (\$1 lost per \$22 wagered) or -4.54%. Historically, Moneyline markets have used slightly less house edge than the typical ATS bet; in practice, the efficient Moneyline ROI is often between 2-3%, implying only about half of the house edge as the ATS bets. Together these two methods provide the alternative yet related definitions of 'efficiency' to be used and tested by our hypothesis.

Next to define: what level of inefficiency may yield profitable results? This is where the Test Center becomes valuable. KARMA defines two different measures of inefficiency, an X-factor and a Y-factor.

The specific definitions and calculations are reserved for those purchasing the full KARMA database; general methodology is outlined in the following sections. The Test Center allows users to easily see how historical results vary with the X- and Y-factors for each sport along with the other user inputs.

Putting it all together provides us a more practical way to define our hypothesis and create our detailed methodology.

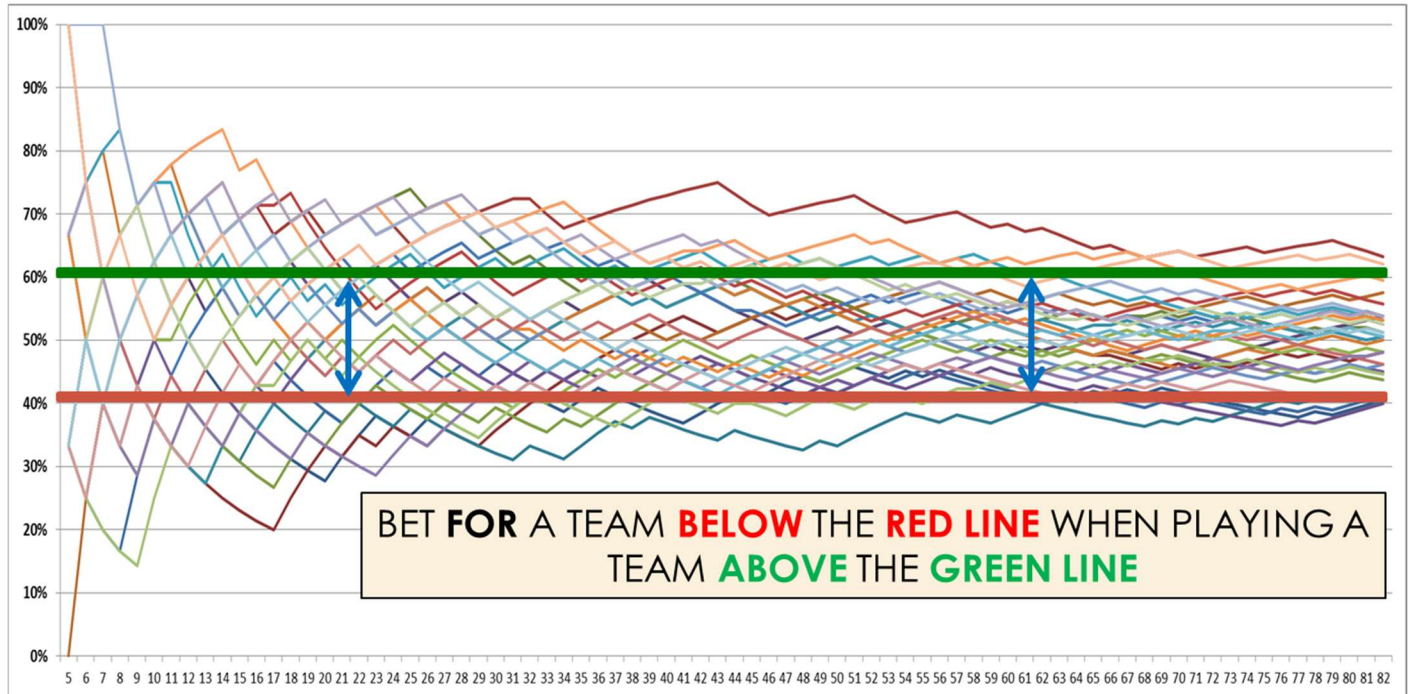
***KARMA tests the premise that betting on teams which have underperformed relative to expectations to-date in the season will have favorable (profitable) lines when playing teams that have overperformed relative to expectations to-date.***

The exact levels of underperformance and overperformance are for the gambler to decide; for this reason, these factors (X and Y) are user inputs to the betting model.

## METHODOLOGY

All of the abstract and hypothesis explanations may be difficult to fully grasp without a proper illustration. The image below is intended to illustrate KARMA's methodology in its simplest form.

Take the time to understand this exhibit. Commentary below provides important context.



The exhibit above charts the 2017 NBA season ATS for each team. 30 lines for 30 teams. Each point represents each team's Spread record ATS entering each game of the season. The regression to the mean that has been mentioned in the previous sections comes alive in this exhibit. However, we should remember that regression to the mean is also a naturally occurring phenomenon. For example, if a team wins only 5 of the first 20 games of the NBA season ATS (25%), then winning half of the remaining games would mean the team finishes 36-46 ATS (44%). This regression from 25% to 44% ATS happens in a perfectly efficient market for the final 62 games of the season and no value would exist when betting on this team *every game* to regress to the mean. Fortunately, gamblers have the advantage of not betting every game while sportsbooks have the liability of posting lines for every game.

This also leads to some of the most important inputs for KARMA: if not every game is valuable, then which games have profitable value? What point in the season is the optimal starting point to bet that markets have reached a profitable level of inefficiency and what are the inefficiency requirements of the opponent? These questions are addressed with the X- and Y-factor inputs along with the starting game input; all of these are inputs that each user can adjust in the Test Center.

If regression to the mean is a naturally occurring phenomenon, how is it that value exists? KARMA has discovered that, beyond some point in the season and when playing an opponent beyond some

threshold of inefficiency, the regression is not entirely natural. The other way regression to the mean occurs is when humans (intentionally or unintentionally) put their thumb on the scale. This is where overweighting small sample sizes creates value. By overweighting small sample sizes, the betting markets incorrectly expect a team's overperformance to-date to continue. This leads to closing lines that provide value for betting on the underperforming team. This is the second type of regression that occurs -- not organically, but through the fallibility of humans. In a truly random experiment (i.e. all independent trials), we cannot beat the spread because past results have no influence on future results. Fortunately, the *dependency* that humans assign from game to game is what creates the inefficiency.

One important item to address is the question of whether profitable inputs in prior seasons will be profitable inputs in future seasons. Of course, a study of regression to the mean must acknowledge that maximizing back-tested results will almost certainly result in diluted (regressed) results going forward. Because of this, the sample inputs and outputs outlined in the Results section deliberately do not choose the maximums. Instead, the Results section summarizes a more balanced set of inputs that attempts to maximize the combination of units and win %.

All of the previous discussion has revolved around measuring each team's individual performance against the spread and total markets as a season progresses. However, the league as a whole also has a universal measure of efficiency (50% Win/Loss, 50% Over/Under, etc.). Notably, the league as a whole is more likely to end near efficiency since the sample size of a full league season is much larger than a single team. In some cases, KARMA considers the league inefficiency for the same variables as for individual teams. For example, determining whether to place an Over bet may require that a) both the X-factor and the Y-factor thresholds are exceeded and b) the league has been some % below efficiency (efficiency being 50%) on Over bets to this point in the season. Considering the additional measurements of league inefficiency does add stability to win rates across seasons for several different markets though it is not needed to stabilize and maximize results for all markets. In fact, it also does not improve results in all markets. This is one area of further future scrutiny.

As mentioned earlier, the definition of the X-factor and the Y-factor would be generally explained. The X-factor in the chart represents the *Difference* between the green line and the red line. The Y-factor represents the *Minimum* inefficiency of the red line. The Demonstration page on karmasports.us shows the user how the X- and Y- factors interact with each other. For totals (overs vs unders), the factor definitions differ slightly due to how the market operates. For spreads or moneyline, bets are preferred when one team has underperformed and the other has overperformed. For totals, bets are preferred when both teams underperform (more unders) or when both teams overperform (more overs). The hypothesis remains unchanged at its core. In all circumstances, KARMA is betting on regression to the mean; the measures and algorithms adjust to incorporate the idiosyncrasies of each market.

The next important item to define in the methodology is how profits and losses are measured. KARMA uses a basic definition of *units*; this assumes that 1 unit is equivalent to a \$1 bet on each game. For spreads and totals, it is assumed that \$1.10 is wagered in order to win \$1. For moneyline bets, it is assumed that \$1.00 is wagered in order to win the closing payoff quoted. Measuring in units provides the simplest conversion for all gamblers. The aggregate win or loss is simply the amount you wager per

game (ex. \$110 to win \$100 or \$1,000 @ +150 to win \$1,500) times the number of units. So a \$1,000 per game gambler would multiply the units KARMA calculates by \$1,000; a +5-unit season would equate to a \$5,000 profit for the season.

In order to enhance KARMA's value, our Test Center provides users with the power to instantly change the key input assumptions to see how value changes. Key inputs for the user to choose include X-factor, Y-factor, and Start Game for each season. These inputs allow the user to view a comprehensive and dynamic range of results instantaneously and split out by season for each sport.

Lastly, the methodology considers each regular season as a single independent market; that is, each game's minimarket also acts as a single data point in the entire season's market. Playoffs present unique challenges and have not been adequately studied; playoffs for all American professional leagues are excluded from the current results. The exception is CFB bowl season, which is now included in Volume II. Since the CFB season is only 1 extra game for virtually all teams, it is considered by KARMA as much closer to an extension of the regular season than a new playoff season.

## DATA AND LIMITATIONS

The website covers.com provides the closing line betting information for all sports, markets, and seasons. Where necessary, other sources were used to manually input closing lines for the small number of missing data points. More than enough historical seasons to pique the interest of sleuths with any number of ideas beyond this paper; this paper focuses on the major leagues since adding their most recent expansion team at least 1 decade prior. There are only a few minor limitations associated with covers.com. First, there are some missing closing lines in the historical data; the missing data in the seasons used by KARMA represent less than 1% of all the data points. The missing lines were assumed by KARMA based upon recent games (for a few baseball series) as well as researching game previews or other available historical betting information (including VegasInsider).

Second, covers.com data does result in slightly different results than vegasInsider.com and other data sources since lines between sources routinely vary by a half point or so. Encouragingly, the difference in results is often small so the ultimate outputs available in the Test Center do not materially vary between the separate sources. This means that if a set of inputs shows consistent value over more than a decade of covers.com data, the same results using vegasInsider.com or other data might differ slightly though results will likely also be positive. To summarize, the Value Graphs in the Appendix do not materially shift or change between data sources.

The data pulled by Covers is assumed to represent the closing line for each game and market. KARMA recognizes that betting opening (or otherwise different) lines may yield different results from the units in the Test Center. The difference is not assumed to be material and, separately, closing lines are generally available for everyone to bet despite occasional exceptions.

Another limitation involves strike-shortened seasons. Strike-shortened seasons are included in Volume II and include all games played each season. This includes the 2012 NBA season as well as the 2013 NHL season. Both are included in historical results which are likely to lead to some small conservatism in measurements of average units over a given period.

Volume I included all sports seasons completed or started before March 2020. Volume II includes all sports leagues started by the end of 2021. Changes in the data since Volume I include the following:

- Seasons were realigned for NBA, EPL and NHL. Now, the year is defined by the *beginning* of each season. Thus, 2021 implies the 2021-2022 season for sports straddling the New Year.
- Strike seasons are now included; partial results are available for the number of games played.
- College Football results now include Bowl Season as a special Week 16 each season.
- College Football now includes an updated 2010 season. Notably, updates include fixing erroneous closing Totals of 27 points for 63 games. CFB Seasons prior to 2010 are excluded in Volume II due to an excessive number of missing closing lines in Covers data.
- All season inputs throughout KARMA workfiles were re-checked as part of developing Volume II. This includes checking all the season inputs, formulas, and adding a few missing lines.

## RESULTS

The exhibit on the next page details KARMA results using one sample set of inputs for each sport and market to illustrate how quickly small or medium value opportunities among several sports can combine into a lucrative betting venture. The exhibit outlines the results of betting on nearly 4,000 sports games over up to 20 seasons for each of 6 major sports since 2002; the total of **+329.4 units won over 3,862 games translates to an average win % of 56.4%**. Units are even better per year beginning in 2005 when MLB and NBA results began; blank entries represent data before what is available from Covers.com.

Notice that there are 3 markets with 'n/a' in all years. This means that we do not recommend using our system to bet in these markets. With humility, we are trying to find consistent success; too much risk is a significant downside and an important part of the considerations when choosing to gamble. Thus, we do not recommend betting MLB Totals, NHL Moneyline, or NHL Totals markets. You *might* be able to break even in these markets using KARMA optimal inputs, but who wants to do that? Not us, based upon the last decade of betting data.

Additionally, we only *lightly* recommend betting on the following markets: MLB Moneyline, NBA Spreads, NBA Totals, and CFB Spreads. We recommend only considering betting on the sport if you really enjoy baseball, basketball, or college football. All of these markets are expected to return less than 2.2 units per year. MLB Moneyline is included here instead of the strongest recommendations because the 2.1 average units shown in the chart required the largest number of games bet across all markets; in other words, it is a bad value. These markets could warrant a small betting bankroll with a tiny expectation. With sincere caution, we do believe there is a slight edge to the gambler, even against the juice; for these markets, we strongly advise not to wager more than fun money.

**Our strongest recommendations are to bet on the following markets: NFL Spread, NFL Total, CFB Totals, EPL Wins, EPL Draws. These markets appear to be capable of achieving at least 2 units per year consistently and to be worth the risks given the win %. In the prior 10 years (2012 thru 2021 seasons), these five markets accounted for +198.0 units won over 1,524 games which is a 58.6% win rate.**

Optimistically, annual results could exceed 15 units per year on average. Notably, the maximum inputs and outputs were not chosen for this exhibit since part of KARMA's study of regression must also acknowledge the irony that maximizing back-tested results will inevitably lead to regression of results when applied going forward. Additional important statistics are documented in the Appendix for each market along with the full Value Graph to show where and how much value exists. The Value Graphs shown are only for a single start point for each Sport; choosing a different starting game/week would result in a different Value Graph.

Different inputs certainly will yield different results. Each user is free to experiment and determine the inputs that match a chosen risk tolerance. One of the only advantages that sports bettors have over the sportsbooks is that the gambler is free to choose only a small subset of games to bet while the sportsbook sets lines and takes bets on every game. This advantage is utilized by KARMA which values winning percentage alongside total units won. We believe these results highlight the strength of the hypothesis amongst many sports and seasons.

## KARMA -- All Sports History -- Sample Results

Year	NFL Spread	NFL Total	All NFL	CFB Spread	CFB Total	All CFB	EPL Win	EPL Draw	All EPL	NBA Spread	NBA Total	All NBA	MLB ML	MLB Total	All MLB	NHL ML	NHL Total	All NHL	Total
2002	2.5	-1.3	1.2																1.2
2003	-0.8	1.2	0.4							-1.7	1.8	0.1			2.3	n/a	2.3	n/a	0.4
2004	1.7	6.9	8.6							-2.4	0.0	-2.4			15.5	n/a	15.5	n/a	8.7
2005	4.7	2.4	7.1							-4.5	1.0	-3.5			9.0	n/a	9.0	n/a	20.5
2006	5.4	3.1	8.5							7.6	-0.2	7.4			15.5	n/a	15.5	n/a	12.6
2007	-7.8	4.0	-3.8							1.2	0.0	1.2			-0.7	n/a	-0.7	n/a	13.2
2008	3.5	9.2	12.7							-4.8	-1.1	-5.9			1.0	n/a	1.0	n/a	3.9
2009	1.2	7.6	8.8							1.5	2.0	3.5			-1.4	n/a	-1.4	n/a	13.1
2010	7.8	-8.2	-0.4							-1.2	0.0	-1.2			8.0	n/a	8.0	n/a	28.9
2011	3.3	4.6	7.9							-4.4	2.9	-1.5			-1.4	n/a	-1.4	n/a	27.0
2012	4.8	5.7	10.5							7.0	-1.3	5.7			3.8	n/a	3.8	n/a	18.2
2013	3.0	-3.1	-0.1							-1.1	0.0	-1.1			0.0	n/a	0.0	n/a	13.7
2014	1.1	4.9	6.0							10.6	0.0	10.6			7.9	n/a	7.9	n/a	31.5
2015	0.2	0.2	0.4							7.0	-0.3	6.7			-0.8	n/a	-0.8	n/a	35.2
2016	6.6	-2.6	4.0							-2.1	1.0	-1.1			4.1	n/a	4.1	n/a	45.4
2017	2.3	9.6	11.9							0.7	-0.1	0.6			-10.1	n/a	-10.1	n/a	-21.8
2018	4.7	0.0	4.7							-0.2	0.0	-0.2			6.4	n/a	6.4	n/a	37.8
2019	3.5	6.3	9.8							-2.3	-0.1	-2.4			0.0	n/a	0.0	n/a	19.8
2020	-1.2	-0.5	-1.7							0.1	0.8	0.9			-8.1	n/a	-8.1	n/a	13.0
2021	-2.6	4.5	1.9																

<b>Avg Units</b>	<b>2.2</b>	<b>2.7</b>	<b>4.9</b>	<b>1.0</b>	<b>5.4</b>	<b>6.5</b>	<b>6.9</b>	<b>3.1</b>	<b>10.0</b>	<b>0.6</b>	<b>0.4</b>	<b>1.0</b>	<b>2.1</b>	<b>0.0</b>	<b>2.1</b>	<b>0.0</b>	<b>0.0</b>	<b>0.0</b>	<b>16.5</b>
X-Factor	34%	34%		20%	55%		41%	69%		10%	16%		20%	n/a	n/a	n/a	n/a	n/a	
Y-Factor	30%	-10%		20%	5%		1%	69%		2%	2%		1%	n/a	n/a	n/a	n/a	n/a	

Start Wk/Gm	3	3	3	3	3	3	21	21	741	900	900	324	900	n/a	991	0	n/a	n/a	<b>Total</b>
Games Bet	328	518	846	517	443	960	433	308	284	40	324	991	0	991	0	0	0	0	<b>3,862</b>

<b>Five Strongest Markets</b>	
Average Units 2012-2021	19.8
Total Units 2012-2021	198.0
Win % 2012-2021	58.6%

<b>Nine Total Markets With Recommendations</b>			
Average Units 2012-2021	22.0	Average Units 2005-2021	18.8
Total Units 2012-2021	219.7	Total Units 2005-2021	319.1
Win % 2012-2021	56.4%	Win % 2005-2021	56.5%
Average Units 2002-2021		Average Units 2002-2021	
Total Units 2002-2021		Total Units 2002-2021	
Win % 2002-2021		Win % 2002-2021	

## WHAT'S NEW IN VOLUME II

Since Volume I, the website for KARMA (karmasports.us) was completed with many new features to enhance the understanding of the willing user. Features of the site now include:

- A web page to demonstrate how the X- and Y-factors interact with each other
- A read-only embedded insight for each sports model showing upcoming picks in addition to season results to date and prior team betting results.
- A carousel of all the Value Graphs for each sport, season, and market
- An updated Test Center Power BI to see results by season based on user inputs

The current outlook of KARMA remains encouraging; after considering the full Results section alongside the following Appendix, there appear to be sustainable profit opportunities across most of the major sports betting markets. All our recommended sports indicate consistently profitable inputs and outputs for each market over at least a decade of results. Particularly in the Appendix Value Graphs, the results change along the X-axis and Y-axis in a generally smooth fashion. This indicates that the results are not the culmination of mining for the few false positives in the dataset but rather that value exists for a large range of inputs with varying magnitude. The additional evidence of KARMA working across multiple sports and across many nuances between markets adds even more creditability to the full story. KARMA is not about discovering false positives through torturing data; it is about illustrating how human fallibility (overreaction and availability bias) is greater than the amount of juice a sportsbook charges.

Going forward, additional sports and playoffs will be considered for the next Volume. The next sport would likely be College Basketball with the MLS being a second option on the horizon. College sports - basketball in particular - present unique challenges since the markets are filled with many more teams yet shorter seasons. However, these aspects may also present uniquely profitable betting opportunities, like the college football Totals market. The MLS is likely not too different from EPL, but the data has not yet been collected. As for playoffs, it seems likely to provide a *tiny* amount of value as it's a small portion of each season. Some additional playoff-specific efficiency measures may also become relevant to predicting outcomes, but all that work is secondary to the core of KARMA which focuses on regular seasons and the major betting markets (especially NFL, CFB, and EPL).

Other future considerations include exploring different variations of the current general methodology. One reason for the release of KARMA is to encourage other users to explore the Test Center and to solicit external suggestions from those who are able to verify and expand upon the current methodology. Regression to the mean can occur in a plethora of markets that can be measured; certainly, a more lucrative methodology exists than this one.

## APPENDIX – DETAILED RESULTS

The following charts in this section illustrate the “Value Graph” for the two main betting markets in each of the sports included in Volume II. Red indicates negative units while green indicates positive units. The inner box border highlights a 20% x 20% (step 2%) grid of the best inputs long term based upon the analysis. The results of these 121 scenarios are summarized in the top right of each snapshot in the Results section, For Select Inputs column. The 25<sup>th</sup>, 50<sup>th</sup>, and 75<sup>th</sup> percentiles of the 121 scenarios are included. While virtually every market lacks efficiency with respect to regression to the mean, not all markets are inefficient enough to profit. This has been discussed in prior sections. The following pages illustrate the sensitivity of results in each Value Graph.

As for betting strategy using these Value Graphs, there is more to the story. The results for most sports show small albeit positive averages for most inputs. However, the results year to year are not nearly as small – often, the standard deviation is larger than the average result which introduces real risk.

This leads to the additional part of the betting strategy that KARMA employs. By looking through these Value Graphs year by year on karmasports.us, you can notice that the value *generally* shifts year to year (meaning slight inverse correlation from the year prior). This implies we should choose one of the worse sets of inputs within the highlighted 121 scenarios each year to choose the next year’s set of inputs. It is not a perfect correlation, and it doesn’t need to be in order to improve results. Given, this is indeed a much more subjective addition to a betting strategy; another way of thinking about it is that this is the art to complement the science.

So, how does this strategy do? Well, slightly better than the best results below. In the recommended betting markets (NFL Spreads & Totals, CFB Totals, EPL Wins & Draws), the first attempt at this subjective addition yielded improved results in 4 of the 5 markets and tied for best in the 5<sup>th</sup> market. Improvements are small but important, averaging up to 1 unit per season and sport.

More testing and experience are needed to refine a subjective part of the equation. Comparably, another part of art vs. science is the timing of placing bets. Choosing to bet before the closing line could yield slight improvements to results but only if line movements are able to be predicted more often than not. These are just some of the long list of considerations for any serious gamblers looking to construct a sustainable betting operation.

The following charts outline results for 6 sports, 2 markets per sport, and all of the seasons available for each market. Results fluctuate across the x-axis by varying the X-factor and similar for the Y-factor.











# NHL

MARKET:  
NHL Moneyline

SEASON SELECTED  
START: 2005  
STOP: 2021

GAMES INCLUDED  
Start Game: 700  
End Game: 1312

SAMPLE INPUTS  
X-factor: 12%  
Y-factor: 12%

RESULTS  
For Sample Inputs: Units 2.2, Wins 103.8, Losses 131.7, 75th Percentile 2.1, Median 1.1, 25th Percentile 0.0

Y-factor (Minimum)	UNITS	1%	3%	5%	7%	9%	11%	13%	15%	17%	19%	21%	23%	25%	27%	29%	31%	33%	35%	37%	39%	41%	43%	45%	47%	49%	51%	53%	55%	57%	59%	61%	63%	65%	67%	69%	71%	73%	75%	77%	79%	81%
-9%	(1.3)	(1.2)	(1.9)	(1.0)	(1.5)	(1.2)	(1.1)	(0.5)	(0.6)	(0.4)	(0.9)	(2.2)	(1.5)	(2.1)	(1.0)	0.4	0.3	(0.6)	0.6	0.7	0.2	(1.1)	(0.6)	0.2	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
-7%	(1.3)	(0.6)	(1.4)	(1.3)	(1.4)	(0.7)	(1.3)	(0.6)	(1.1)	(0.3)	(0.7)	(1.5)	(1.0)	(2.2)	(1.0)	0.2	0.6	(0.3)	0.8	0.8	0.3	(1.1)	(0.6)	0.2	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
-5%	(2.5)	(0.9)	(1.4)	(1.7)	(1.9)	(0.6)	(1.5)	(0.8)	(0.5)	(0.8)	(1.2)	(2.0)	(1.5)	(2.7)	(1.6)	(1.1)	0.5	(0.4)	0.8	0.7	0.3	(1.1)	(0.5)	0.2	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
-3%	(2.5)	(0.9)	(1.0)	(1.0)	(1.6)	(0.2)	(0.6)	(0.2)	(0.3)	(0.7)	(1.1)	(1.9)	(1.6)	(2.5)	(1.4)	(0.0)	0.7	(0.3)	0.9	0.9	0.4	(1.0)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
-1%	(1.0)	1.1	1.0	1.5	0.5	1.7	1.0	0.4	(0.0)	(0.8)	(0.9)	(1.8)	(1.6)	(2.5)	(1.4)	0.1	0.9	(0.3)	0.9	0.9	0.4	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
1%	(0.2)	2.3	1.1	2.0	1.0	2.6	1.6	0.2	(0.1)	(0.5)	(0.8)	(1.7)	(1.5)	(2.4)	(1.1)	0.2	0.8	(0.3)	0.9	0.9	0.4	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
3%	(0.1)	2.2	1.3	2.7	1.7	3.3	1.9	0.2	(0.1)	(0.1)	(0.5)	(1.3)	(1.5)	(2.5)	(1.1)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
5%	(1.2)	1.9	0.9	2.7	1.6	3.4	1.8	0.0	(0.2)	(0.2)	(0.3)	(1.2)	(1.4)	(2.4)	(1.1)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
7%	(1.3)	2.3	1.4	3.3	1.8	3.7	1.9	0.2	(0.1)	(0.0)	(0.2)	(1.0)	(1.3)	(2.4)	(1.1)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
9%	(1.9)	2.0	1.3	3.2	1.9	3.8	2.0	0.3	0.0	0.0	(0.1)	(1.0)	(1.3)	(2.4)	(1.1)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
11%	(2.6)	1.3	0.9	3.4	2.1	3.9	2.1	0.2	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
13%	(2.2)	1.6	1.1	3.5	2.2	3.9	2.1	0.2	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
15%	(2.3)	1.6	1.1	3.5	2.0	3.8	2.1	0.2	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
17%	(1.9)	2.0	1.3	3.6	2.1	3.9	2.1	0.2	0.0	0.0	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
19%	(1.8)	2.2	1.4	3.6	2.1	3.9	2.1	0.2	0.0	0.0	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
21%	(1.6)	2.5	1.4	3.6	2.0	3.8	2.0	0.1	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
23%	(1.5)	2.6	1.4	3.5	2.0	3.8	2.0	0.1	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
25%	(1.4)	2.6	1.4	3.5	2.0	3.8	2.0	0.1	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
27%	(1.3)	2.7	1.4	3.6	2.0	3.8	2.0	0.1	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
29%	(1.3)	2.7	1.4	3.6	2.0	3.8	2.0	0.1	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
31%	(1.3)	2.7	1.4	3.6	2.0	3.8	2.0	0.1	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
33%	(1.3)	2.7	1.4	3.6	2.0	3.8	2.0	0.1	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
35%	(1.3)	2.7	1.4	3.6	2.0	3.8	2.0	0.1	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
37%	(1.3)	2.7	1.4	3.6	2.0	3.8	2.0	0.1	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
39%	(1.3)	2.7	1.4	3.6	2.0	3.8	2.0	0.1	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
41%	(1.3)	2.7	1.4	3.6	2.0	3.8	2.0	0.1	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
43%	(1.3)	2.7	1.4	3.6	2.0	3.8	2.0	0.1	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
45%	(1.3)	2.7	1.4	3.6	2.0	3.8	2.0	0.1	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
47%	(1.3)	2.7	1.4	3.6	2.0	3.8	2.0	0.1	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
49%	(1.3)	2.7	1.4	3.6	2.0	3.8	2.0	0.1	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2	0.8	(0.4)	0.9	1.0	0.5	(1.1)	(0.4)	0.3	0.6	0.1	0.8	1.0	0.4	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	
51%	(1.3)	2.7	1.4	3.6	2.0	3.8	2.0	0.1	(0.0)	(0.0)	(0.1)	(1.1)	(1.4)	(2.5)	(1.2)	0.2																										